DIFFERENTIAL EQUATIONS

EXERCISE 2.10

Problems solved by;

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PARAMETERS the solution of the equation.

$$y'' + Py' + Qy = r(x). \quad \text{—— (1)}$$

where P, Q and r are functions of x which are continuous on some interval I. The method gives a particular solution of (1) on I in the form.

$$y_p(x) = -y_1 \int \frac{y_2 r(x)}{W} dx + y_2 \int \frac{y_1 r(x)}{W} dx \quad \text{—— (2)}$$

where y_1 and y_2 form a basis of the solutions of homogeneous equations.

$$y'' + Py' + Qy = 0 \quad \text{—— (3)}$$ corresponding to (1) and W is the wronskian of y_1 and y_2.

PROOF

Suppose that linearly independent solutions of (3) are $y = y_1$ and $y = y_2$. Then.

$$y_h = c_1 y_1 + c_2 y_2$$

with c_1 and c_2 arbitary constants. We replace these arbitary constants by unknown functions $u_1(x)$ & $u_2(x)$, and require that

$$y_p = u_1 y_1 + u_2 y_2 \quad \text{—— (4)}$$

be the particular solution of (1)

In order to determine u_1 and u_2 we need two conditions. One condition is that (4) must satisfy (1). A second condition we can impose arbitarily

differentiating (4) w.r.t x we get.

$$y_p' = u_1' y_1 + u_2' y_2 + u_1 y_1' + u_2 y_2'$$

If we further differentiate this y_p'' contains u_1'' and u_2''. To avoid 2nd derivatives of u_1 and u_2. we set.

$$u_1' y_1 + u_2' y_2 \equiv 0 \quad \text{—— (5)}$$

$y_p' = u_1 y_1' + u_2 y_2'$

$\Rightarrow y_p'' = u_1' y_1' + u_1 y_1'' + u_2' y_2' + u_2 y_2''$.

substituting for y_p, y_p' and y_p'' into ①

$\Rightarrow (u_1' y_1' + u_1 y_1'' + u_2' y_2' + u_2 y_2'') + P(u_1 y_1' + u_2 y_2')$

$+ Q(u_1 y_1 + u_2 y_2) = r(x)$

$\Rightarrow u_1(y_1'' + P y_1' + Q y_1) + u_2(y_2'' + P y_2' + Q y_2)$.

$+ u_1' y_1' + u_2' y_2' = 0$.

$y_1'' + P y_1' + Q y_1 = 0$ because y_1 is a sol of ③

Similarly $y_2'' + P y_2' + Q y_2 = 0$

$\therefore u_1' y_1' + u_2' y_2' = r(x)$ — ⑥.

Taking ④ and ⑤ together we have.

$u_1 y_1' + u_2' y_2' = 0$

$u_1 y_1' + u_2' y_2' = r(x)$.

Solving these, we have

$$\left.\begin{aligned} u_1' &= \frac{-y_2 r(x)}{y_1 y_2' - y_1' y_2} = \frac{-y_2 r(x)}{W} \\ u_2' &= \frac{y_1 r(x)}{y_1 y_2' - y_1' y_2} = \frac{y_1 r(x)}{W} \end{aligned}\right\} — ⑦$$

In ⑦ $W \neq 0$ since y_1 and y_2 are linearly Independent Solutions of ③

by Integrating we get.

$u_1 = \int -\frac{y_2 r(x)}{W} dx$ & $u_2 = \int \frac{y_1 r(x)}{W} dx$.

$$\boxed{\therefore y_p = -y_1 \int \frac{y_2 r(x)}{W} dx + y_2 \int \frac{y_1 r(x)}{W} dx}$$

parameters because here we have change the parameter (i.e c_1 and c_2 by u_1 and u_2).

EXERCISE 2.10

Find General Solution of Non-homogeneous Eqs

① $y'' - 4y' + 4y = \frac{e^{2x}}{x}$. —— ①

Sol For y_h, we have the characteristic eq of the corresponding homogeneous linear eq as

$\lambda^2 - 4\lambda + 4 = 0 \Rightarrow \lambda^2 - 2\lambda - 2\lambda + 4 = 0 \Rightarrow \lambda(\lambda-2) - 2(\lambda-2) = 0$

$\Rightarrow \lambda = 2, 2.$ (double roots)

Hence.

$y_h = (c_1 + c_2 x)e^{2x}.$

so $y_1 = e^{2x}$ & $y_2 = xe^{2x}$.

$\therefore W = \begin{vmatrix} e^{2x} & xe^{2x} \\ 2e^{2x} & e^{2x} + 2xe^{2x} \end{vmatrix} = e^{4x} - \cancel{2xe^{4x}} - \cancel{2xe^{4x}}$
$= e^{4x}.$

So Particular Sol of ① is.

$\Rightarrow y_p = -y_1 \int \frac{y_2\, r(x)}{W}\, dx + y_2 \int \frac{y_1\, r(x)}{W}\, dx$

$\Rightarrow y_p = -e^{2x} \int \frac{\cancel{x}\cancel{e^{2x}}\left(\cancel{e^{2x}}/\cancel{x}\right)}{\cancel{e^{4x}}}\, dx + xe^{2x} \int \frac{\cancel{e^{2x}}\left(\cancel{e^{2x}}/x\right)}{\cancel{e^{4x}}}\, dx$

$\Rightarrow y_p = -xe^{2x} + xe^{2x}\ln x.$

$\therefore y = y_h + y_p$
$= (c_1 + c_2 x - x + x\ln x)e^{2x}$ Ans.

Sol For y_h, we have.

$\lambda^2+9=0 \Rightarrow \lambda = \pm 3i.$ (CASE - III)

so $y_h = A\cos 3x + B\sin 3x.$

Q2

$$W = \begin{vmatrix} \cos 3x & \sin 3x \\ -3\sin 3x & 3\cos 3x \end{vmatrix} = 3\cos^2 3x + 3\sin^2 3x. \\ = 3.$$

$$\therefore y_p = -\cos 3x \int \frac{\sin 3x \cdot \sec 3x}{3} dx + \sin 3x \int \frac{\cos 3x \cdot \sec 3x}{3} dx$$

$$\Rightarrow y_p = \frac{1}{9}\cos 3x \ln\cos 3x + \frac{1}{3} x \sin 3x.$$

Ans

③ $y'' + 2y' + y = e^{-x}\cos x$

Sol For y_h, we have

$\lambda^2 + 2\lambda + 1 = 0.$

$\Rightarrow \lambda^2 + \lambda + \lambda + 1 = 0 \Rightarrow \lambda(\lambda+1) + 1(\lambda+1) = 0$

$\Rightarrow \lambda = -1, -1.$ (CASE II)

hence $y_h = (c_1 + c_2 x)e^{-x}.$

$$W = \begin{vmatrix} e^{-x} & xe^{-x} \\ -e^{-x} & e^{-x} - xe^{-x} \end{vmatrix} = e^{-2x} - \cancel{xe^{-2x}} + \cancel{xe^{-2x}} \\ = e^{-2x}.$$

$$\therefore y_p = -e^{-x}\int \frac{x\cancel{e^{-x}} \cdot \cancel{e^{-x}}\cos x}{\cancel{e^{-2x}}}dx + xe^{-x}\int \frac{\cancel{e^{-x}}\,\cancel{e^{-x}}\cos x}{\cancel{e^{-2x}}}dx$$

$$\Rightarrow y_p = -e^{-x}\left[x\sin x - \int \sin x\, dx\right] + xe^{-x}\sin x.$$

$$\Rightarrow y_p = -\cancel{xe^{-x}\sin x} - e^{-x}\cos x + \cancel{xe^{-x}\sin x}$$

$$\Rightarrow y_p = -e^{-x}\cos x.$$

$$\therefore y = y_h + y_p = (c_1 + c_2 x - \cos x)e^{-x}.$$

Ans

For y_h, we have $\lambda^2+9=0 \Rightarrow \lambda = \pm 3i$.

$\Rightarrow y_h = A\cos 3x + B\sin 3x$.

Now

Q4

$$W = \begin{vmatrix} \cos 3x & \sin 3x \\ -3\sin 3x & 3\cos 3x \end{vmatrix} = 3.$$

$$\therefore y_p = -\cos 3x \int \frac{\sin 3x \cdot \operatorname{cosec} 3x}{3}\,dx + \sin 3x \int \frac{\cos 3x \operatorname{cosec} 3x}{3}\,dx$$

$$\Rightarrow y_p = -\frac{x}{3}\cos 3x + \frac{1}{9}\sin 3x \ln \sin 3x.$$

& Hence $y = A\cos 3x + B\sin 3x - x/_3 \cos 3x + 1/_9 \sin 3x \ln \sin 3x$ Ans.

(5) $y'' - 2y' + y = e^x/x^3$.

For y_h we have

$\lambda^2 - 2\lambda + 1 = 0 \Rightarrow \lambda^2 - \lambda - \lambda + 1 = 0 \Rightarrow \lambda(\lambda-1) - 1(\lambda - 1) = 0$

$\Rightarrow \lambda = 1, 1$ (CASE II)

$\therefore y_h = (c_1 + c_2 x)e^x$.

&

$$W = \begin{vmatrix} e^x & xe^x \\ e^x & e^x + xe^x \end{vmatrix} = e^{2x} + xe^{2x} - xe^{2x} = e^{2x}.$$

$$\therefore y_p = -e^x \int \frac{\cancel{e^x}\, \cancel{e^x}/x^{3^2}}{\cancel{e^{2x}}}\,dx + xe^x \int \frac{\cancel{e^x e^x}/x^3}{\cancel{e^{2x}}}\,dx$$

$$\Rightarrow y_p = -e^x \frac{1}{-x} + \cancel{x}e^x \frac{1}{-2x^{\cancel{2}}} = \frac{e^x}{x} - \frac{e^x}{2x}.$$

$$\therefore y = y_h + y_p = \left(c_1 + c_2 x + \frac{1}{x} - \frac{1}{2x}\right)e^x.$$

$$\Rightarrow y = \left(c_1 + c_2 x + \frac{1}{2}x\right)e^x \text{ Ans}$$

(6) $y'' - 4y' + 5y = e^{2x}\operatorname{cosec} x$

For y_h, we have

$\lambda^2 - 4\lambda + 5 = 0 \Rightarrow \lambda = \dfrac{4 \pm \sqrt{16-20}}{2} = \dfrac{4 \pm 2i}{2}$

$\Rightarrow \lambda = 2 \pm i$ (CASE III)

$\therefore y_h = e^{2x}(A\cos x + B\sin x)$.

Now

$$W = \begin{vmatrix} e^{2x}\cos x & e^{2x}\sin x \\ 2e^{2x}\cos x - e^{2x}\sin x & 2e^{2x}\sin x + e^{2x}\cos x \end{vmatrix}$$

$\Rightarrow W = 2e^{4x}\sin x\cos x + e^{4x}\cos^2 x - 2e^{4x}\sin x\cos x$
$+ e^{4x}\sin^2 x$

$\Rightarrow W = e^{4x}.$

$\therefore y_p = -e^{2x}\cos x\int \frac{e^{2x}\sin x\, e^{2x}\operatorname{cosec} x}{e^{4x}}dx + e^{2x}\sin x\int \frac{e^{2x}\cos x\, \frac{e^{2x}}{\sin x}}{e^{4x}}dx$

$\Rightarrow y_p = -xe^{2x}\cos x + e^{2x}\sin x\ \ln\sin x.$

$\therefore y = y_h + y_p = (A\cos x + B\sin x - x\cos x + \sin x\ln\sin x)e^{2x}$

Ans

⑦ $(D^2 - 2D + 1)y = 3x^{3/2}e^x$

for y_h, we have.

$\lambda^2 - 2\lambda + 1 = 0 \Rightarrow \lambda^2 - \lambda - \lambda + 1 = 0 \Rightarrow \lambda(\lambda-1) - 1(\lambda-1) =$

$\Rightarrow \lambda = 1, 1.$ (CASE II)

$\therefore y_h = (c_1 + c_2 x)e^x.$

$W = \begin{vmatrix} e^x & xe^x \\ e^x & e^x + xe^x \end{vmatrix} = e^{2x} + xe^{2x} - xe^{2x} = e^{2x}.$

$\therefore y_p = -e^x\int \frac{xe^x\, 3x^{3/2}e^x}{e^{2x}}dx + xe^x\int \frac{e^x\, 3x^{3/2}e^x}{e^{2x}}dx$

$\Rightarrow y_p = -\frac{6e^x}{7}\cdot x^{7/2} + \frac{6e^x x^{7/2}}{5}$

$\therefore y = y_h + y_p = \left(c_1 + c_2 x + \left(-\frac{6}{7} + \frac{6}{5}\right)x^{7/2}\right)e^x.$

$\Rightarrow y = \left(c_1 + c_2 x + \frac{12}{35}x^{7/2}\right)e^x$ $\quad \frac{-30+42}{35}$

Ans

⑧ $(D^2 + 6D + 9)y = 16e^{-3x}/(x^2+1)$

for y_h, we have.

$\lambda^2 + 6\lambda + 9 = 0 \Rightarrow \lambda^2 + 3\lambda + 3\lambda + 9 = 0.$

$\Rightarrow \lambda(\lambda+3) + 3(\lambda+3) \Rightarrow \lambda = -3, -3$ (CASE II)

$\therefore y_h = (c_1 + c_2 x)e^{-3x}.$

$$\begin{vmatrix} \cdots & \cdots \\ -3e^{-3x} & e^{-3x}-3xe^{-3x} \end{vmatrix} = e^{-6x} - 3xe^{-6x} + 3xe^{-6x} = e^{-6x}.$$

$$\therefore y_p = -e^{-3x}\int \frac{xe^{-3x}\, 6e^{-3x}/(x^2+1)}{e^{-6x}}\,dx + xe^{-3x}\int \frac{e^{-3x}\cdot e^{-3x}/(x^2+1)}{e^{-6x}}\,dx$$

$$\Rightarrow y_p = -\frac{1}{2}e^{-3x}\int \frac{2x}{x^2+1}\,dx + xe^{-3x}\int \frac{dx}{x^2+1}$$

$$\Rightarrow y_p = -\frac{1}{2}e^{-3x}\ln|x^2+1| + xe^{-3x}\tan^{-1}x.$$

$$\therefore y = y_h + y_p = \left(c_1 + c_2 x - \tfrac{1}{2}\ln|x^2+1| + x\tan^{-1}x\right)e^{-3x}$$

Ans

⑨ $(D^2+4D+4)y = 2e^{-2x}/x^2.$

For y_h, we have.

$\lambda^2 + 4\lambda + 4 = 0 \Rightarrow \lambda^2 + 2\lambda + 2\lambda + 4 = 0.$

$\Rightarrow \lambda(\lambda+2) + 2(\lambda+2) \Rightarrow \lambda = -2, -2.$ (CASE II)

hence $y_h = (c_1 + c_2 x)e^{-2x}.$

Also

$$W = \begin{vmatrix} e^{-2x} & xe^{-2x} \\ -2e^{-2x} & e^{-2x} - 2xe^{-2x} \end{vmatrix} = e^{-4x} - 2xe^{-4x} + 2xe^{-4x} = e^{-4x}.$$

$$\therefore y_p = -e^{-2x}\int \frac{xe^{-2x}\cdot 2e^{-2x}/x^2}{e^{-4x}} + xe^{-2x}\int \frac{e^{-2x}\, 2e^{-2x}/x^2}{e^{-4x}}\,dx$$

$$\Rightarrow y_p = -2e^{-x}\ln|x| + 2xe^{-2x}\left(-\frac{1}{x}\right)$$

$$\Rightarrow y_p = -2e^{-x}\ln x - 2e^{-2x}$$

$$\therefore y = y_h + y_p = (c_1 + c_2 x)e^{-2x} - (2\ln x + 2)e^{-2x}.$$

Ans

⑩ $(D^2+2D+2)y = 4e^{-x}\sec^3 x$

For y_h, we have.

$\lambda^2 + 2\lambda + 2 = 0 \Rightarrow \cancel{\lambda^2 + \lambda + \lambda + 2}$

$$\Rightarrow \lambda = \frac{-2 \pm \sqrt{4-8}}{2} = -1 \pm i \quad \text{(CASE - III)}$$

$$\therefore y_h = e^{-x}(A\cos x + B\sin x).$$

$$\begin{vmatrix} \cdots \\ -e^{-x}\cos x - e^{-x}\sin x & -e^{-x}\sin x + e^{-x}\cos x \end{vmatrix}$$

$\Rightarrow W = -e^{-2x}\sin x\cos x + e^{-2x}\cos^2 x + e^{-2x}\sin x\cos x + e^{-2x}\sin^2 x$

$\Rightarrow W = e^{-2x}.$

$\therefore y_p = -e^{-x}\cos x \int \frac{e^{-x}\sin x \cdot 4e^{-x}\sec^3 x}{e^{-2x}}dx + e^{-x}\sin x \int \frac{e^{-x}\cos x \sec^3 x}{e^{-2x}}dx$

$\Rightarrow y_p = -e^{-x}\cos x \int \sec x \cdot \sec x \tan x\, dx + e^{-x}\sin x \int \sec^2 x\, dx.$

$\Rightarrow y_p = -e^{-x}\cos x \int \sec x \frac{d}{dx}\sec x\, dx + e^{-x}\sin x \tan x.$

$\Rightarrow y_p = -e^{-x}\cos x \frac{\sec^2 x}{2} + e^{-x}\sin x \tan x.$

$\Rightarrow y_p = -\frac{1}{2}e^{-x}\sec x + e^{-x}\sin x \tan x.$

or $y_p = -e^{-x}\cos x \int \sec^2 x \tan x\, dx + e^{-x}\sin x \int \sec^2 x\, dx.$

$\Rightarrow y_p = -e^{-x}\cos x \frac{\tan^2 x}{2} + e^{-x}\sin x \tan x.$

$\Rightarrow y_p = -\frac{1}{2}e^{-x}\sin^2 x \sec x + e^{-x}\sin^2 x \sec x.$

$\Rightarrow y_p = \frac{1}{2}e^{-x}\sin^2 x \sec x.$

$\therefore y = \left(A\cos x + B\sin x + \frac{1}{2}\sin^2 x \sec x\right)e^{-x}$ Ans

Find G.S of the following Non-homogeneous Euler Chauchy equations.

(11) $x^2y'' - 4xy' + 6y = 21x^{-4}$ —— (1)

for y_h, we have

$m^2 + (a-1)m + b = 0.$

$\Rightarrow m^2 + (-5)m + 6 = 0$

$\Rightarrow m^2 - 5m + 6 = 0$

$\Rightarrow m^2 - 3m - 2m + 6 = 0 \Rightarrow m(m-3) - 2(m-3) = 0$

$\Rightarrow m = 2, \quad m = 3.$

For y_p, we make the eq (1) standard.

i.e $y'' - \frac{4}{x}y' + \frac{6}{x^2}y = 21x^{-6}$.

Now.

$$y_p = -x^2\int \frac{x^3 \cdot 21x^{-6}}{W}dx + x^3\int \frac{x^2 \cdot 21x^{-6}}{W}dx$$

Now $W = \begin{vmatrix} x^2 & x^3 \\ 2x & 3x^2 \end{vmatrix} = 3x^4 - 2x^4 = x^4$.

$$\therefore y_p = -x^2\int \frac{x^3 21 x^{-6}}{x^4}dx + x^3\int \frac{x^2 21x^{-6}}{x^4}dx$$

$$\Rightarrow y_p = -21x^2\int x^{-7}dx + 21x^3\int x^{-8}dx$$

$$\Rightarrow y_p = -21x^2 \cdot \frac{x^{-6}}{-6} + 21x^3\frac{x^{-7}}{-7}$$

$$\Rightarrow y_p = \frac{21}{6}x^{-4} - \frac{21}{7}x^{-4}$$

$$\Rightarrow y_p = \frac{21}{42}x^{-4} = \frac{1}{2}x^{-4}$$

$$\therefore y = y_h + y_p = c_1x^2 + c_2x^3 + \frac{1}{2}x^{-4}$$

Ans

(12) $x^2y'' - xy' = (3+x)x^3e^x$. —— (1)

For y_h, we have.

$m^2 + (-1-1)m + 0 = 0$

$\Rightarrow m^2 - 2m = 0 \Rightarrow m(m-2) = 0 \Rightarrow m = 0, 2$

(CASE – I)

$\therefore y_h = c_1x^0 + c_2x^2$.

Also

$W = \begin{vmatrix} 1 & x^2 \\ 0 & 2x \end{vmatrix} = 2x$.

i.e $y'' - \frac{y'}{x} = (3+x)xe^x$.

$$\therefore y_p = -\int \frac{x^2(3+x)\cancel{x}e^x}{2\cancel{x}}dx + x^2\int \frac{(3+x)\cancel{x}e^x}{2\cancel{x}}dx$$

$$\Rightarrow y_p = -\frac{1}{2}\int (3x^2+x^3)e^x dx + \frac{x^2}{2}\int (3+x)e^x dx$$

$$\Rightarrow y_p = -\frac{1}{2}\left[(3x^2+x^3)e^x - \int e^x(6x+3x^2)dx\right] + \frac{x^2}{2}\left[(3+x)e^x - \int e^x dx\right]$$

$$\Rightarrow y_p = -\frac{1}{2}\left[(3x^2+x^3)e^x - \left[e^x(6x+3x^2) - \int e^x(6+6x)dx\right]\right] + \frac{x^2}{2}\left[(3+x)e^x - e^x\right]$$

$$\Rightarrow y_p = -\frac{1}{2}(3x^2+x^3)e^x + \frac{e^x}{2}(6x+3x^2) - \frac{1}{2}\left[e^x(6+6x) - \int 6e^x dx\right]$$
$$+ \frac{x^2}{2}(3+x)e^x - \frac{x^2}{2}e^x\Big]$$

$$\Rightarrow y_p = -\frac{1}{2}(3x^2+x^3)e^x + \frac{e^x}{2}(6x+3x^2) - \frac{1}{2}e^x(6+6x) + 3e^x$$
$$+ \frac{x^2}{2}(3+x)e^x - \frac{x^2}{2}e^x.$$

$$\Rightarrow y_p = \left(-\frac{3}{2}x^2 - \frac{1}{2}x^3 + 3x + \frac{3}{2}x^2 - 3 - 3x + 3 + \frac{3}{2}x^2 + \frac{x^3}{2} - \frac{x^2}{2}\right)e^x$$

$$\Rightarrow y_p = 3x^2e^x.$$

~~$y = y_h + y_p = c_1 + (c_2 + 3x^2)$~~

$$\Rightarrow y = y_h + y_p = c_1 + c_2x^2 + 3x^2e^x.$$ Ans

(13) $4x^2y'' + 8xy' - 3y = 7x^2 - 15x^3$ —— ①

For y_h, we have

$x^2y'' + 2xy' - \frac{3}{4}y = \frac{7}{4}x^2 - \frac{15}{4}x^3 \to 0$

and hence

$m^2 + (2-1)m - 3/4 = 0.$

$\Rightarrow m^2 + m - 3/4 = 0. \Rightarrow m = \frac{-1 \pm \sqrt{1+3}}{2}$

$\Rightarrow m = \frac{-1 \pm 2}{2} = \frac{1}{2}, -\frac{3}{2}.$

$\therefore y_h = c_1x^{1/2} + c_2x^{-3/2}.$

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For y_p, we have to make (1) standard.

i.e $y'' + \frac{2}{x}y' - \frac{3}{4x^2}y = \frac{7}{4} - \frac{15}{4}x$ —— (2).

$$\therefore y_p = -y_1\int \frac{y_2\left(\frac{7-15x}{4}\right)}{W}dx + y_2\int \frac{y_1\left(\frac{7-15x}{4}\right)}{W}dx.$$

$$W = \begin{vmatrix} x^{1/2} & x^{-3/2} \\ \frac{1}{2}x^{-1/2} & -\frac{3}{2}x^{-5/2} \end{vmatrix} = -\frac{3}{2}x^{-2} - \frac{1}{2}x^{-2} = -2x^{-2}.$$

$$\therefore y_p = -\frac{x^{1/2}}{4}\int \frac{x^{-3/2}(7-15x)}{-2x^{-2}}dx + \frac{x^{-3/2}}{4}\int \frac{x^{1/2}(7-15x)}{-2x^{-2}}dx.$$

$$\Rightarrow y_p = +\frac{x^{1/2}}{8}\int x^{1/2}(7-15x)\,dx - \frac{x^{-3/2}}{8}\int x^{5/2}(7-15x)\,dx$$

$$\Rightarrow y_p = \frac{x^{1/2}}{8}\int (7x^{1/2} - 15x^{3/2})\,dx - \frac{x^{-3/2}}{8}\int (7x^{5/2} - 15x^{7/2})\,dx$$

$$\Rightarrow y_p = \frac{x^{1/2}}{\cancel{8}\,4}\left[\frac{7x^{3/2}}{3/\cancel{2}} - \frac{15x^{5/2}}{5/\cancel{2}}\right] - \frac{x^{-3/2}}{\cancel{8}\,4}\left[\frac{\cancel{7}x^{7/2}}{\cancel{7}/\cancel{2}} - \frac{\cancel{15}\,5\,x^{9/2}}{\cancel{9}\,3/\cancel{2}}\right].$$

$$\Rightarrow y_p = \frac{x^{1/2}}{4}\left[\frac{7}{3}x^{3/2} - 3x^{5/2}\right] - \frac{x^{-3/2}}{4}\left[x^{7/2} - \frac{5}{3}x^{9/2}\right].$$

$$\Rightarrow y_p = \frac{7}{12}x^2 - \frac{3}{4}x^3 - \frac{x^2}{4} + \frac{5}{12}x^3$$

$$\Rightarrow y_p = \frac{1}{3}x^2 - \frac{1}{3}x^3.$$

$$\therefore y = y_h + y_p = c_1x^{1/2} + c_2x^{-3/2} + \frac{1}{3}x^2 - \frac{1}{3}x^3$$

Ans

P- T- O →

Q#14 $(x^2D^2 - 4xD + 6)y = 7x^4\sin x.$

for y_h, we have.

$m^2 + (-4-1)m + 6 = 0 \Rightarrow m^2 - 5m + 6 = 0.$

$\Rightarrow m^2 - 3m - 2m + 6 = 0 \Rightarrow m(m-3) - 2(m-3) = 0.$

$\Rightarrow m = 2, 3.$

hence $y_h = c_1x^2 + c_2x^3.$

Also

$$W = \begin{vmatrix} x^2 & x^3 \\ 2x & 3x^2 \end{vmatrix} = 3x^4 - 2x^4 = x^4.$$

Hence $y_p = -x^2\int \dfrac{x^3 \cdot \frac{7x^4\sin x}{x^2}}{x^4}dx + x^3\int \dfrac{x^2 \cdot \frac{7x^4\sin x}{x^2}}{x^4}dx$

$\Rightarrow y_p = -x^2\int 7x\sin x\,dx + x^3\int 7\sin x\,dx.$

$\Rightarrow y_p = -x^2\left[-7x\cos x - \int -7\cos x\,dx\right] + x^3(-7\cos x)$

$\Rightarrow y_p = \cancel{7x^3\cos x} - 7x^2\sin x - \cancel{7x^3\cos x}.$

$\Rightarrow y_p = -7x^2\sin x.$

$\therefore y = y_h + y_p = c_1x^2 + c_2x^3 - 7x^2\sin x.$ Ans

Q#15 $(x^2D^2 - 2xD + 2)y = x^3\cos x$

for y_h, we have.

$m^2 - 3m + 2 = 0 \Rightarrow m^2 - 2m - m + 2 = 0 \Rightarrow m(m-2) - 1(m-2)$

$\Rightarrow m = 1, 2.$ (CASE I)

$\therefore y_h = c_1x^1 + c_2x^2.$

Also

$$W = \begin{vmatrix} x & x^2 \\ 1 & 2x \end{vmatrix} = 2x^2 - x^2 = x^2.$$

hence.

$y_p = -x\int \dfrac{x^2 \cdot \frac{x^3\cos x}{x^2}}{x^2}dx + x^2\int \dfrac{x \cdot \frac{x^3\cos x}{x^2}}{x^2}dx.$

$\Rightarrow y_p = -x\int x\cos x\,dx + x^2\int \cos x\,dx.$

$\Rightarrow y_p = -x\left[x\sin x - \int(\sin x)dx\right] + x^2\sin x.$

$\Rightarrow y_p = -\cancel{x^2\sin x} + x(-\cos x) + \cancel{x^2\sin x}$

$\Rightarrow y_p = -x\cos x.$

Ans

(16) $(x^2D^2 + xD - 1)y = 1/x^2$

Sol: for y_h, we have,

$$m^2 + (1-1)m - 1 = 0 \Rightarrow m^2 - 1 = 0 \Rightarrow m = \pm 1.$$

Q16

$$\therefore y_h = c_1 x + c_2 x^{-1}.$$

Also $$W = \begin{vmatrix} x & x^{-1} \\ 1 & -1/x^2 \end{vmatrix} = -1/x - 1/x = -2/x$$

$$\therefore y_p = -x\int \frac{\frac{1}{x}\cdot 1/x^4}{-2/x}\,dx + 1/x\int \frac{x \cdot 1/x^4}{-2/x}\,dx.$$

$$\Rightarrow y_p = +\frac{x}{2}\int \frac{1}{x^4}\,dx - \frac{1}{2x}\int \frac{1}{x^2}\,dx.$$

$$\Rightarrow y_p = \frac{x}{2}\frac{x^{-3}}{-3} - \frac{1}{2x}\frac{x^{-1}}{-1}$$

$$\Rightarrow y_p = \frac{-1}{6x^2} + \frac{1}{2x^2} = \frac{1}{3x^2}$$

$$\therefore y = y_h + y_p = c_1 x + c_2/x + 1/3x^2 \quad \text{Ans}$$

(17) $(x^2D^2 + xD - 9)y = 48x^5$

for y_h, we have.

$$m^2 + (1-1)m - 9 = 0 \Rightarrow m = \pm 3.$$

$$\therefore y_h = c_1 x^3 + c_2 x^{-3}.$$

Also

$$W = \begin{vmatrix} x^3 & x^{-3} \\ 3x^2 & -3x^{-4} \end{vmatrix} = -3x^{-1} - 3x^{-1} = -6/x$$

$$\therefore y_p = -x^3\int \frac{x^{-3}\cdot 48x^5}{-6/x}\,dx + x^{-3}\int \frac{x^3\cdot 48x^5}{-6/x}\,dx.$$

$$\Rightarrow y_p = \frac{48x^3}{6}\int x\,dx - \frac{48}{6}\int x^7\,dx$$

$$\Rightarrow y_p = 8x^3\left[\frac{x^2}{2}\right] - 8x^{-3}\left[\frac{x^8}{8}\right].$$

$$\Rightarrow y_p = 4x^5 - x^5 = 3x^5.$$